Reg. No. ....

Name ....

(2 pages)

K 6124

SECOND YEAR B.Sc. DEGREE EXAMINATION, APRIL/MAY 2005

Part III—Mathematics

Paper II—SUBSIDIARY FOR STATISTICS (Main)

Time : Three Hours | Maximum : 110 Marks

Not more than 22 marks will be awarded for each unit.
Each question carries 6 marks.

Unit I

1. (a) A particle moves along a curve whose parametric equations are given by $x = e^t$, $y = \cos 5t$, $z = \sin 5t$ where t is the time taken. Find the velocity and acceleration at time $t = 0$.

   (b) If $\phi = xy + xy^2 + xz^2 + yz - 1$; find $\nabla\phi$ at (1, 2, 1).

2. (a) If $\bar{r}$ is the position vector of a moving point P (x, y, z) find $\nabla\left(\frac{1}{r}\right)$ where $r = |\bar{r}|$.

   (b) Find the unit normal vector to the surface $xyz = 1$ at (1, 1, 1).

3. If $\overline{F} = (x^2 - 2yz)\bar{i} + (y^2 - 2xz)\bar{j} + (z^2 - 2xy)\bar{k}$ show that $\overline{F}$ is irrotational and find its scalar potential.

4. If $\bar{f} = 4xy\,\bar{i} + y^3\,\bar{j}$ evaluate $\int_C \bar{f}\cdot d\bar{r}$ along the curve C in xy plane ; $y = x^2$ from (0, 0) to (1, 1).

5. If $\overline{A} = 2xy\,\bar{i} + yz^2\,\bar{j} + xz\,\bar{k}$ and S is a rectangular parallelopiped bounded by $x = 0, y = 0, z = 0, x = 1, y = 2, z = 3$ evaluate $\iint_S \overline{A}\cdot\bar{n}\,ds$.

Unit II

6. Obtain a differential equation of all straight lines which are at a fixed distance from origin.

7. (a) Solve the equation $\frac{x\,dy - y\,dx}{x^2 + y^2} + dx = 0$.

   (b) Solve $\frac{dy}{dx} = \frac{3x - 2y}{2x - 3y}$.

8. (a) Find the general and particular solution of $(D^2 + 4D - 12)y = e^{3x}$ where $D = \frac{d}{dx}$.

   (b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = \log x$.

9. Solve the simultaneous differential equations $\frac{dx}{dt} - y = t$ and $\frac{dy}{dt} + x = 1$.

10. Solve $(y + z)\,p + (z + x)\,q = x + y$ with usual notations for p and q.

Unit III

11. Show that the set of all matrices of type $\begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ (where p and q are integers) under matrix addition forms a group.

12. Show that the set S = {0, 1} (mod 2) is a field with respect to addition and multiplication (mod 2).

13. Define a vector space V over a field F and give an illustrative example.

14. Determine whether or not the vectors (1, – 2, 1), (2, 1, – 1), (7, – 4, 1) are linearly dependent in $\mathbb{R}^3$.

15. Obtain an orthonormal basis for $\mathbb{R}^3$ by applying Gram-Schmidt orthogonalization process to the basis {(1, 1, 1), (0, 1, 1), (0, 0, 1)} for $\mathbb{R}^3$.

Unit IV

16. Define Linear Transformation T on a vector space V. Explain its structure using an example.

17. Let E be an idempotent linear transformation on a vector space V. If M is the range of E and N the null space of E, show that :

    (a) x is in M if and only if $Ex = x$.

    (b) V is the direct sum of M and N.

18. Let $T : R^2 \to R^2$ be the linear map given by $T(x, y) = (x + 4y, 2x - 3y)$. Find the matrix representation of T with respect to to basis {(1, 3), (2, 5)}.

19. Transform the quadratic form $4x^2 + y^2 - 8z^2 + 4xy - 4xz + 8yz$ to the diagonal form $aX^2 + bY^2 + cZ^2$.

20. Define a quadratic form. Find the matrix of the quadratic form :

$$Q(x) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3.$$

Unit V

21. State the Cauchy–Riemann equations and illustrate the theorem for $f(z) = z^2$.

22. Define Harmonic functions in xy plane. Obtain the harmonic conjugate of $u(x, y) = y^3 - 3x^2y$.

23. Find the linear fractional transformation that maps $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$.

24. Expand $\cos z$ into a Taylors series about the point $z = \frac{\pi}{2}$.

25. Evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$ using the method of residues.