Reg. No. ....

Name ....

(3 pages)

K 6122

SECOND YEAR B.Sc. DEGREE EXAMINATION, APRIL/MAY 2005

Part III—Mathematics

Paper II—SUBSIDIARY FOR PHYSICS (MAIN)

Time : Three Hours — Maximum : 110 Marks

[Maximum marks that can be earned from each unit is 22.]

Unit I

1. Find the acceleration of the particle which moves along the curve $x = 2 \sin 3t$, $y = 2 \cos 3t$, $z = 8t$ at $t = \pi/2$.

(4 marks)

2. If $\overline{A} = 5t^2 \bar{i} + t\bar{j} - t^3 \bar{k}$ and $\overline{B} = 3t^2 \bar{i} - 2t\bar{k}$, find $\frac{d}{dt}\left(\overline{A} \times \overline{B}\right)$. (5 marks)

3. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $|\bar{r}| = r$ show that $\nabla \log r = \frac{\bar{r}}{r^2}$. (4 marks)

4. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, show that (a) div $\bar{r} = 3$; (b) curl $\bar{r} = 0$. (3 + 3 = 6 marks)

5. Determine the constant "a" so that the vector $\overline{F} = (x + 3y)\bar{i} + (y - 2z)\bar{j} + (x + az)\bar{k}$ is solenoidal.

(5 marks)

6. By the Divergence theorem, evaluate $\iint_S \overline{F} \cdot \bar{n}\, ds$, where $\overline{F} = 4xz\,\bar{i} - y^2 \bar{j} + yz\,\bar{k}$ and S is the surface of the cube bounded by $x = 0, x = 1$; $y = 0, y = 1$; $z = 0, z = 1$.

(6 marks)

Unit II

7. Let G = {– 9, – 8, . . . – 1, 0, 1, 2, . . . 8, 9}. Is G a subgroup of the additive group of integers. Give reason for your answer.

(4 marks)

8. Give an example of :

   (a) Finite commutative ring.

   (b) An infinite commutative ring.

   (c) An infinite non-commutative ring.

(3 marks)

Turn over

9. Prove that the set F = {0, 1, 2, 3, 4} forms a field under addition modulo 5 and multiplcation modulo 5.

(5 marks)

10. Consider the linear map $f : \mathbb{R}^3 \to \mathbb{R}^2$, given by $f(x, y, z) = (x, y)$. Find the Kernel of f. (4 marks)

11. Define a vector space and show that $\mathbb{R}^3$ is a vector space. (5 marks)

12. Show that the vectors (1, 1, 1) (1, 2, 3) and (2, – 1, 1) in $\mathbb{R}^3$ are linearly independent. (4 marks)

13. Determine the eigenvalues and the corresponding eigenvectors of the matrix $\begin{pmatrix} 0 & 1 \\ 4 & 3 \end{pmatrix}$. (5 marks)

Unit III

14. Solve $(D^2 - 2D + 1)y = e^{2x} - \cos x$. (5 marks)

15. Solve $(D^2 + 4D + 5)y = x^2$. (5 marks)

16. Solve $\frac{dx}{dt} + 2x - 3y = t$; $\frac{dy}{dt} - 3x + 2y = e^{2t}$. (6 marks)

17. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$. (5 marks)

18. Eliminating the constants "l and m" from $lx^2 + my^2 + z^2 = 1$, obtain a potential differential equation of order 1.

(3 marks)

19. Find the general solution of :

    (a) $xp + yq = z$.

    (b) $yzp + xzp = xy$.

    where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(6 marks)

Unit IV

20. Solve $x' + x = t$ given $x(0) = 0$. (4 marks)

21. Obtain the general solution in a series of powers of x of the equation :

$$\left(x - x^2\right)\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 2y = 0.$$

(5 marks)

22. Show that $\int_{-1}^{+1}\left[P_n(x)\right]^2 dx = \frac{2}{2n+1}$. (5 marks)

23. Using $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x) \cdot t^n$, show that :

    (a) $P_n(-1) = (-1)^n$.

    (b) $P_{2n}(0) = \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n \cdot n!}$.

(6 marks)

24. Define $J_n(z)$ and show that $J_{-n}(z) = (-1)^n J_n(z)$. (5 marks)

25. Show that $J_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}} \cos z$. (5 marks)

Unit V

26. Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function $w = u + iv$.

(6 marks)

27. Show that an analytic function $f(z) = u + iv$ is constant if its modulus is constant. (5 marks)

28. Expand $\frac{1}{z^2 - 3z + 2}$ in powers of z.

    (a) When $1 < |z| < 2$.

    (b) When $|z| > 2$.

(5 marks)

29. Evaluate $\int_C \frac{2z^2 + z}{z^2 - 1} dz$, where "C" is the circle $|z| = 2$. (4 marks)

30. Find the residues of $\frac{z+1}{z^2(z-2)}$ at its poles. (5 marks)

31. Show that for $a > 1$, $\int_0^{\pi} \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$. (5 marks)