Reg. No..... (Pages : 2) **K 5102**

Name.....

SECOND YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2005

Part III—Group I—Mathematics

Paper II—ALGEBRA AND CALCULUS

Time : Three Hours Maximum : 65 Marks

Maximum 13 *marks for each unit.*

Unit I

1. Prove that the least divisor of a composite number is a prime number. (4 marks)
2. Find the number and sum of the divisors of 7128. (4 marks)
3. If n is any natural number, prove that $n\ (n + 1)\ (n + 2)$ is divisible by 6. (4 marks)
4. Show that the remainder when 2^{46} is divided by 47 is 1. (4 marks)
5. Show that the 8th power of any number is of the form 17 m or 17 m $\pm$ 1. (4 marks)

Unit II

6. Solve $4x^4 - 8x^3 + 7x^2 + 2x - 2 = 0$ given that $1 + i$ is a root. (5 marks)
7. Solve $x^4 + 2x^3 - 25x^2 - 26x + 120 = 0$ given that the product of two of its roots is 8. (5 marks)
8. Find the sum of the fourth powers of the roots of $x^3 - x - 1 = 0$. (5 marks)
9. Find the number and position of the real roots of the equation $x^5 - 5x + 1 = 0$. (5 marks)

Unit III

10. Find the value of $\int_0^2 (x + 4)\, dx$ as limit of sums. (5 marks)
11. Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. (5 marks)
12. Find the integration, the volume of a right circular cone of base radius r and height h. (5 marks)
13. Prove that $\lceil 1/2 = \sqrt{\pi}$. (5 marks)

Turn over

Unit IV

14. Compute the inverse of the matrix :

$$\begin{bmatrix} 4 & -2 & -1 \\ 1 & 1 & -1 \\ -1 & 2 & 4 \end{bmatrix}.$$

(4 marks)

15. Find the rank of the matrix :

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}.$$

(5 marks)

16. Test for consistency and solve :

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 4 \\ x + 4y + 9z &= 6. \end{aligned}$$

(6 marks)

17. Using Cayley Hamilton theorem, show that $A^3 - 6A^2 + 11A - 6I = 0$ where :

$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence find A^{-1}.

(5 marks)

Unit V

18. Find the Fourier series to represent $f(x)$ in $[-\pi, \pi]$ given $f(x) = x + x^2$ in $-\pi < x < \pi$.

(4 marks)

19. Obtain the Fourier series for the function f defined by $f(x) = x - \pi$ if $-\pi < x < 0$ and $f(x) = \pi - x$ if $0 < x < \pi$. Use it to get a series with sum π^2.

(5 marks)

20. Show that $\cos x = \frac{8}{\pi} \sum_{n=1}^{8} \frac{n \sin 2nx}{4n^2 - 1}$ if $0 < x < \pi$. (5 marks)

21. Find the Fourier series for $|x|$ in $[-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

(6 marks)