Reg. No..... (Pages : 2) **K 5108**

Name .....

FINAL YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2005

Part III—Group I—Mathematics

Paper VIII—Elective—GRAPH THEORY

Time : Three Hours Maximum : 65 Marks

Unit I

A maximum of 10 *marks can be earned from this unit.*

1. Define a graph. Give an example of a graph with 5 vertices and 7 edges. Define isomorphism between two graphs. Define an Eulerian graph and give an example.

(2 + 1 + 2 + 2 + 1 = 8 marks)

2. Prove that the number of vertices of odd degree in a graph is always even.

(2 + 2 = 4 marks)

3. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(2 marks)

4. Explain the problem of 4 multicoloured cubes and give a method of solving it graphically.

(3 + 3 = 6 marks)

Unit II

A maximum of 15 *marks can be earned from this unit.*

5. Show that a connected graph G is Eulerian iff all vertices of G are of even degree. (7 marks)

6. Define union and intersection of two graphs G_1 and G_2. Define decomposition of a graph G into two subgraphs G_1 and G_2. Give examples for each.

(1 + 1 + 1 + 1 +1 + 1 = 6 marks)

7. Draw a connected graph G such that G-e is disconnected for each edge e of G. (2 marks)

8. Show that in a complete graph with n vertices, there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

(5 marks)

9. Prove that a connected graph G remains connected after removing an edge e_i from G if and only if e_i is in some circuit of G.

(5 marks)

Unit III

A maximum of 20 *marks can be earned from this unit.*

10. Define a tree. Prove that a connected graph on n vertices is a tree iff it has $n - 1$ edges.

(1 + 4 + 4 = 9 marks)

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11. Define center of a tree and show that every tree has either one or two centres. (6 marks)

12. Draw all trees on four labelled vertices. (4 marks)

13. Define spanning tree and show that every connected graph has at least one spanning tree.

(1 + 4 = 5 marks)

14. Define a branch and a chord of a spanning tree T of a graph G. (2 + 2 = 4 marks)

15. Show that in any tree with two or more vertices, there are at least two pendant vertices. What is the maximum number of pendant vertices in a tree with n vertices where n is a positive integer ?

(4 + 3 = 7 marks)

Unit IV

A maximum of 10 *marks can be earned from this unit.*

16. Prove that in a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cutset.

(4 marks)

17. Show that every cutset in a connected graph G must contain at least one branch of every spanning tree.

(3 marks)

18. Prove that in a tree, every vertex of degree at least two is a cutvertex. (3 marks)

19. Define vertex connectivity and edge connectivity. Show that the edge connectivity of a graph G can not exceed the degree of the vertex with the smallest degree in G.

(1 + 1 + 2 = 4 marks)

20. Prove that every connected graph with three or more vertices has at least two vertices which are not cutvertices.

(4 marks)

Unit V

A maximum of 10 *marks can be earned from this unit.*

21. Show that the complete graph on 5 vertices is non-planar. (5 marks)

22. State Euler's formula for planar graphs. Prove that Kuratowski's second graph is non-planar.

(2 + 5 = 7 marks)

23. Every region of a simple Planar graph (with n vertices and e edges) embedded in a plane is bounded by K edges. Show that $e = \frac{k(n-2)}{k-2}$.

(5 marks)