Reg. No. .....

(2 pages)

K 5106

Name .....

FINAL YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2005

Part III—Group I—Mathematics

Paper VI—COMPLEX VARIABLES

Time : Three Hours

Maximum : 65 Marks

The maximum marks for each unit is 13.

Unit I

1. Show that $2\left\{\left|z_1^2\right| + \left|z_2^2\right|\right\} = \left|z_1 + z_2\right|^2 + \left|z_1 - z_2\right|^2$. (4 marks)
2. Find the equation of the circle having the line joining z_1 and z_2 as diameter. (4 marks)
3. Show that the function $f(z) = xy + iy$ is every where continuous but is not analytic. (4 marks)
4. Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z. (4 marks)
5. Show that the analytic function with constant modulus is a constant. (4 marks)

Unit II

6. Show that cross-ratio is invariant under a bilinear transformation. (5 marks)
7. Under the mapping $w = z^2$, show that the family of circles $|w - 1| = c$ is transformed into the family of lemniscates $|z - 1|\,|z + 1| = c$. (4 marks)
8. Define a conformal mapping. Is the mapping $w = e^z$ conformal throughout $\mathbb{C}$? (3 marks)
9. Find the invariant points of the transformation $w = \dfrac{3z - 4}{z - 1}$. (3 marks)
10. Discuss the transformation $w = \dfrac{z + 1}{z}$. (5 marks)

Unit III

11. State and prove Cauchy's integral formula. (5 marks)
12. Evaluate $\int_C \dfrac{dz}{z^2\left(z^2 + 4\right)}$ where C is the countour (a) $|z| = \dfrac{3}{2}$, (b) $|z - 2i| = 3$. (5 marks)
13. State and prove Liouville's theorem. (5 marks)
14. Evaluate $\int_C e^z \Big/ (z + 2)(z + 1)^2\, dz$ where C is the circle $|z| = 3$. (3 marks)
15. State Cacuchy's inequality. (2 marks)

Turn over

Unit IV

16. Expand $z\,e^{2z}$ in a Taylor Series about $z = -1$ and determine the region of convergence. (4 marks)

17. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n} (z - 2i)^n$. (3 marks)

18. State and prove Taylor's theorem. (5 marks)

19. Expand in Laurent series $\frac{1}{z(z-1)^2}$ at $z = +1$. (4 marks)

20. Obtain the Laurent's series of the function $f(z) = \frac{1}{(z-1)(z-3)}$ valid in the region :

    (a) $1 < |z| < 3$.

    (b) $|z| > 3$.

    (4 marks)

Unit V

21. Find all zeros of the following functions :—

    (a) $z \tan z$.

    (b) $(z^2 - 1)(z^2 - 3z + 2)$.

    (4 marks)

22. Find the singularities of the following functions and classify the singularities :—

    (a) $z\,e^{1/z}$.

    (b) $\sin\left(\frac{1}{(1-z)}\right)$.

    (4 marks)

23. State and prove Residue theorem. (4 marks)

24. Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$. (4 marks)

25. Evaluate $\int_0^{\infty} \frac{dx}{1 + x^4}$. (4 marks)